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IV.

ON THE SOLAR MOTION IN SPACE.

By Truman Henry Safford.

Presented, Nov. 11, 1874.

I. It was first suggested by Sir W. Herschel that the sun with its planets is moving towards the constellation Hercules. That the sun moves seems to be a necessary consequence of the law of universal gravitation; but in what direction was not noticed until Herschel pointed it out, upon rather imperfect evidence.

Bessel doubted that the evidence proved any thing certain with reference to this motion of the sun, but began the collection of more facts. This was continued by Argelander, who first fairly solved the problem, by Pond, the Struves, and others; and discussions by Lundahl, O. Struve, Galloway, and Mädler agreed in method with Argelander, and their results were substantially in accordance with his and with Herschel's.

The following are the various right ascensions and declinations of the points towards which the solar motion is directed, according to these various authorities:—

Herschel	$A = 257°$	$D = +25°$
	$= 245\ 52'.5$	$+49\ 38'$
Argelander . . .	$= 259\ 51.8$	$+32\ 29.1$
Lundahl	$= 252\ 24.4$	$+14\ 26.1$
O. Struve	$= 261\ 23.1$	$+37\ 35.7$
Galloway	$= 257\ \ 4.4$	$+34\ 18.1$

Of the four latest determinations, that of Argelander is based upon the most stars. It is true that he finally employed but 390; but these are selected from a larger list of 560, and these again from the 3,222 of the Fundamenta Astronomiæ, as giving certain evidence of proper motion: while O. Struve employed 392 in all.

Argelander's 390 were observed by himself for the modern determination, and with very great accuracy.

Lundahl's investigation is based upon the 147 stars which Pond had observed, and Argelander not, and which exhibited proper motions exceeding 0″.09 yearly.

Galloway used 81 stars from the southern hemisphere; this investigation alone employs an old authority (Lacaille) independent of Bessel's Bradley.

A later investigation by Mädler takes account of the motions of all the stars (3,222 in number) found in the Fundamenta Astronomiæ of Bessel, as observed by Bradley near the epoch 1755. Mädler employs the same method of deducing the apex of solar motion as Argelander and the others; and his proper motions of the stars were obtained by himself from a discussion of all available observations. The task seems to have been too vast for his strength, as his proper motions are often erroneous; nor does he appear to have subjected the observations to the careful criticism which Argelander has always employed. Hence the errors of observation are often treated as proper motion by him, as happens more uniformly in the thoroughly worthless values contained in the British Association Catalogue; a work which has served a good purpose as a working-list, but in other respects has retarded rather than advanced our knowledge of star-places.

Mädler's results are: $A = 261° 38'.5$, $D = +39° 53'.6$.

All these discussions of the solar motion are based upon the apparent proper motions of the stars: these are caused by the real motions of both sun and stars; the latter element is considered to follow the law of casual error, and takes the place of error of observation in the discussion. The special method consists in assuming the pole of solar motion, determining from that in what direction each star should move if its own motion were zero, and comparing this direction with the observed; then, by the application of conditional equations and the method of least squares, corrections are found to the assumed pole.

II. To this process Sir George Airy objects: —

1. That we are not sure of even the rude accuracy of the first assumption.

2. Therefore our differential equations will not hold good.

3. And, if they did, the resulting error might change *per saltum* from +179° to —179° by a small change in the assumption; rendering it far from clear whether the method of least squares could properly be thus applied.

He suggests the employment of rectangular co-ordinates of the various stars, assuming their relative distances according to magnitude, as estimated by W. Struve. In this way he obtained the following positions of the point towards which the motion is directed: —

$$\left.\begin{array}{ll} A = 256^\circ\ 54', & D = 39^\circ\ 29' \\ \quad = 261\ \ 29, & \quad = 26\ \ 44 \end{array}\right\}, \text{ with different modes of treatment;}$$

and his assistant Dunkin, from more stars, the following: —

$$\left.\begin{array}{ll} A = 261^\circ\ 14'.0, & D = 32^\circ\ 55'.0 \\ \quad = 263\ \ 43.9, & \quad = 25\ \ \ 0.5 \end{array}\right\}, \text{ with Airy's two modes of treatment.}$$

The proper motions here employed are the Rev. R. Main's, derived from a direct comparison of the positions of the 12 and 6 year Greenwich Catalogues with Bessel's Bradley, and are consequently (upon the whole) more accurate than Mädler's, though not including so many stars. A good many of them are substantially equal to the similar values used by Argelander or Lundahl. Airy employed 113 and Dunkin 1,167 stars: the former set are those whose proper motions are the largest.

Kovalski has used a similar method, employing Mädler's proper motions, and assuming all the stars to be equidistant from the sun. His results were in general similar to those of previous investigators.

III. But unfortunately both Airy and Dunkin express still some doubt about the reality of the result; for the sum of squares of the observed motions is diminished by only one twenty-fifth part on the introduction of the solar motion; so that, although the accordance of the various positions of the pole of motion is gratifying, the suggestion is made by Dunkin that some improvement in our knowledge both of stellar proper motions and stellar distances is yet necessary.

IV. Meanwhile Argelander has within a few years collected the materials for discussing the proper motions of several hundred stars, mostly of the smaller magnitudes, which on this account had been previously overlooked. As the matter now stands, there are between fifty and sixty known stars whose annual proper motion is greater than 1″: not much more than half of these are visible to the naked eye. So that, it seems, magnitude is a very uncertain criterion of proper motion; and all the evidence shows that it is also an uncertain test of distance from us.

V. Some years ago I made the attempt to determine the solar motion from the 250 stars investigated by Argelander in the first part of the 8th volume of the Bonn observations. Of all these, or nearly all, the proper motion was certain; of many it was large; six or seven of them only had been previously used in the problem.

In studying their motions, I grouped them by tens, assuming those to be equally distant whose proper motions in arc were nearest equal. My result for the apex was not very different from those which preceded; but there appeared indications that for each such group the average proper motion was inversely proportional to the average distance, or, in other words, that our assumptions of star-distances ought to depend upon proper motion.

VI. The method I used was substantially Airy's; but in working out the problem I employed a little device like one I have used to simplify the study of planetary orbits. I selected the point whose right ascension was 259° 50′.8, and whose declination was 32° 29′.1, as a point in the positive direction of the axis of *Z;* that of *X* I located in the equator, in right ascension 349° 50′.8, and that of *Y* in right ascension 79° 50′.8 and declination 57° 30′.9. Thus, so far as Argelander's apex is correct, the average stellar motion will be negative, along the axis of *Z;* all other motions will, upon the whole, counteract each other.

VII. So I found it, at least approximately; and, in addition, that the mean *Z*'s for each group of ten were as nearly equal as their probable errors would lead us to expect, taking, as before stated, the proper motion of each star as the measure of the reciprocal of its distance.

VIII. The present paper is for the purpose of showing this relation with respect to the stars earlier investigated by Argelander, Lundahl, and Galloway.

The details of O. Struve's paper are not published, and his stars are largely identical with those of the others; Mädler's investigations have suffered, as before mentioned, from the many errors in his proper motions, which I have detected over large areas of the heavens, but which it will be impossible to get rid of without an investigation costing enormous labor. I propose to continue these investigations from time to time, as material for them accumulates: the great difficulty in the whole matter arises from the extremely unsystematic way in which it has been the fashion to observe star-places, and work up their results. So soon as the great co-operative zones now in progress are completed, much more will be known regarding proper motions; so that for the present what I give here may suffice. I should have added my previous results, but have not been able to get the papers from Chicago, owing to the illness of my assistant, who has had charge of them.

IX. Assuming for the moment that the apex of solar motion is known, and employing Argelander's notation; if now we compute

$$\begin{aligned}
\cos\chi &= -\sin D \sin\delta + \cos D \cos\delta \cos(\alpha - A), \\
\sin\psi' &= \frac{\cos D \sin(\alpha - A)}{\sin\chi}, \\
\Delta\varsigma \sin\psi &= \Delta\alpha \cos\delta, \\
\Delta\varsigma \cos\psi &= \Delta\delta;
\end{aligned}$$

(where α, δ, are the star's right ascension and declination,

A, D, the like co-ordinates of the apex,

χ the star's distance from the apex,

ψ' the angle of position at the star of the great circle passing through star and apex,

$\Delta\alpha$, $\Delta\delta$, the star's annual proper motion in right ascension and declination,

$\Delta\varsigma$ the same in arc of a great circle,

ψ the angle of position in which the star appears to move): then will each star give an equation $\zeta \sin\chi = r\,\Delta\varsigma \cos(\psi' - \psi)$.

Here ζ is the annual solar motion, and r the star's distance from the sun.

X. We shall now proceed as follows: —

Grouping together stars whose proper motions are nearly equal, and making *within such a group* $r\,\Delta\varsigma = 1$, we shall find from each group a value of ζ expressed in terms of $r\,\Delta\varsigma$, or $\frac{\zeta}{r\,\Delta\varsigma}$. If these values of $\frac{\zeta}{r\,\Delta\varsigma}$ for *widely* different values of $\Delta\varsigma$ are nearly equal, — as they proved to be in the preliminary investigation, — we may conclude that $r\,\Delta\varsigma$ is nearly constant over a wide range of values of $\Delta\varsigma$; or, in other words, that star-distances are on the whole inversely proportional to proper motions.

As the values of $\sin\chi$ vary greatly, it is proper to find $\frac{\zeta}{r\,\Delta\varsigma}$ from each group by least squares; and I have done so.

The formula used was $\frac{\zeta}{\Delta\varsigma} = \frac{\Sigma\,[\sin\chi \cos(\psi' - \psi)]}{\Sigma \sin^2\chi}$

GALLOWAY'S SOUTHERN STARS.

Group.	No. Stars.	Greatest and least $\Delta\varsigma$	Value of $\frac{\zeta}{r\Delta\varsigma}$
I.	10	3″581 — 0″540	0.532
II.	10	0.479 — 0.246	0.856
III.	10	0.244 — 0.194	0.723
IV.	10	0.190 — 0.150	0.723
V.	10	0.149 — 0.127	0.765
VI.	15	0.125 — 0.085	0.851

Galloway has 81 stars: I have omitted 12 of the 16 whose ancient places he derives from Bradley, as these are either in Argelander's or Lundahl's investigations, and not uniform with the rest as to the old position.

The 4 of these which are not contained in Argelander's or Lundahl's lists have proper motions ($\Delta\varsigma$) varying from 0″.122 to 0″.101, and give $\frac{\zeta}{r\Delta\varsigma} = 0.992$.

ARGELANDER'S STARS.

Group.	No. Stars.	$\Delta\varsigma$		Value of $\frac{\zeta}{r\Delta\varsigma}$
Class I.				
I.	10	5″13	1″33	0.666
II.	11	1.32	1.01	0.848
Class II.				
I.	10	0″98	0″79	0.447
II.	10	0.77	0.69	0.718
III.	10	0.68	0.57	0.389
IV.	10	0.57	0.53	0.611
V.	10	0.53	0.48	0.886

ARGELANDER'S STARS (*continued*).

Class III.

Group.	No. Stars.	$\Delta\varsigma$		Value of $\frac{\zeta}{r\Delta\varsigma}$
I.	9	0. 49	0. 46	0.909
II.	10	0. 46	0. 42	0.762
III.	10	0. 42	0. 40	0.962
IV.	10	0. 40	0. 38	0.907
V.	10	0. 38	0. 36	0.757
VI.	10	0. 36	0. 35	0.429
VII.	10	0. 35	0. 33	0.533
VIII.	10	0. 33	0. 32	0.684
IX.	10	0. 32	0. 30	0.647
X.	10	0. 30	0. 29	0.478
XI.	10	0. 29	0. 28	1.006
XII.	10	0. 28	0. 27	0.547
XIII.	10	0. 27	0. 26	0.455
XIV.	10	0. 25	0. 25	0.816
XV.	10	0. 25	0. 24	0.753
XVI.	10	0. 24	0. 23	0.719
XVII.	10	0. 23	0. 22	0.277
XVIII.	10	0. 22	0. 21	0.146
XIX.	10	0. 21	0. 21	0.312
XX.	10	0. 21	0. 20	0.613
XXI.	10	0. 20	0. 19	0.897
XXII.	10	0. 19	0. 18	0.637
XXIII.	10	0. 18	0. 17	0.257
XXIV.	10	0. 17	0. 17	0.530
XXV.	10	0. 17	0. 16	0.524
XXVI.	10	0. 16	0. 15	0.669
XXVII.	10	0. 15	0. 14	0.956
XXVIII.	10	0. 14	0. 13	0.589
XXIX.	10	0. 13	0. 12	0.309
XXX.	10	0. 12	0. 11	+0.735
XXXI.	10	0. 11	0. 10	—0.067
XXXII.	9	0. 10	0. 09	+0.869

Note.—I omitted from Class III., Group I., the star C. A., No. 167, which properly belongs in Class II., Group II., and would there change $\frac{\zeta}{r\Delta\varsigma}$ to 0.762. The values of this quantity were computed with two decimals in $\sin^2\chi$ and $\sin\chi$ as $(\psi'-\psi)$, and are not always sure to the third.

The stars investigated by Lundahl show less accordance. There are more of the stars with small proper motions (0″.12 — 0″.09), which appear to move not in accordance with the hypothesis of solar motion: I attribute this, in part at least, to the errors in Pond's and Bradley's observations, which, for this series, are less perfectly eliminated than for Argelander's.

LUNDAHL'S STARS.

Group.	$\Delta \varsigma$	No. Stars.	Value of $\frac{\zeta}{r \Delta \varsigma}$
I.	1″21 — 0″25	7	1.045
II.	0.23 — 0.19	10	0.474
III.	0.19 — 0.18	10	0.448
IV.	0.18 — 0.16	10	0.683
V.	0.16 — 0.15	10	0.839
VI.	0.15 — 0.14	10	0.740
VII.	0.14 — 0.13	10	0.622
VIII.	0.13 — 0.13	10	0.721
IX.	0.13 — 0.12	10	0.280
X.	0.12 — 0.11	10	0.371
XI.	0.11 — 0.11	10	0.072
XII.	0.11 — 0.11	10	0.315
XIII.	0.11 — 0.10	10	0.042
XIV.	0.10 — 0.09	10	0.514
XV.	0.09 — 0.09	10	0.695

For values of proper motion between 5″.13 and 0″.13 annually there is no constant deviation of the value $\frac{\zeta}{r \Delta \varsigma}$ from 0.666, or nearly $\frac{2}{3}$; but from 0″.13 to 0″.09, — that is, the smallest proper motions thought safe to use in the original investigations, — its value is about 0.46, with unusual fluctuations. The last value of each set is by some chance larger than 0.666.

The proper motion of each star being taken as unity, it will be diminished in every case by the amount

$$\cos^2 (\psi' - \psi) - \left[\cos (\psi' - \psi) - \frac{\zeta \sin \chi}{r \Delta \varsigma}\right]^2.$$

The first term, $\cos^2 (\psi' - \psi)$, denotes the square of the observed proper motion of each star in the direction of the great circle passing through the star and the apex of solar motion. This is the direction in which alone the proper motion is affected by the solar motion. The second term $\left[\cos (\psi' - \psi) - \frac{\zeta \sin \chi}{r \Delta \varsigma}\right]^2$ is the square of this component after the solar motion $\frac{\zeta}{r \Delta \varsigma}$ has been subtracted, projected upon the line passing through the star, in the plane passing through the star, apex, and the eye, perpendicular to the line of sight. The difference of these two squares is the proportionate part of the square of the proper motion thus allowed for; and for each star, making $\frac{\zeta}{r \Delta \varsigma} = \varkappa$, a constant is equal to

$$2 \varkappa \sin \chi \cos (\psi' - \psi) - \varkappa^2 \sin^2 \chi.$$

For all the stars we shall need to take the sum

$$2\,\varkappa\,\Sigma\,(\sin\chi\cos(\psi'-\psi))-\varkappa^2\,\Sigma\sin^2\chi.$$

But,

$$\frac{\Sigma\,(\sin\chi\cos(\psi'-\psi))}{\Sigma\sin^2\chi}=\varkappa,\text{ or }\Sigma\,(\sin\chi\cos(\psi'-\psi))=\varkappa\,\Sigma\sin^2\chi;$$

hence, for n stars the sum of squares n of proper motions, each taken equal to unity, will be diminished by $\varkappa^2\,\Sigma\sin^2\chi$, or about $0.24\,n$: as $\varkappa^2=0.38$, and the average $\sin^2\chi$ is about 0.64.

The conclusions I would draw from the investigations so far are these: —

1. In studying the solar motion, the distances must be assumed with reference to the amount of proper motion, and (approximately) in inverse proportion to it.

2. The smaller proper motions (0″.13 or less annually) need careful study at this time.

3. There is some hope of using the solar motion as a sort of base to advance our knowledge of stellar distances.

4. The parallaxes of all stars whose parallax exceeds 1″ annually (about 60 in number) should be systematically determined by a co-operative arrangement.